Benha University Faculty of Engineering – Shoubra Department of Energy and Sustainable Energy Course: Mathematics 3 Code: EMP 201	Final Exam Date : December 24, 2016 Answer **All** questions Duration: 2 hours
• The exam consists of one page	• No. of questions: 4 Total Mark: 40

Question 1	
(a)Find the first derivatives of the function : $f(x,y,z) = x.\sinh x + 2y^4 + \ln z$	4
(b)Find the envelope of the curves : $x^2 + (y-a)^2 = 4a.$	4
(c)Determine the extrema of the function : $f(x,y) = \mathrm{x}^2 - 3\mathrm{y}^2 - 8\mathrm{x} + 12\mathrm{y}.$	4
Question 2	
(a) Find $\nabla\mathrm{F}$ where : $F = x^4 + x.\sin y + y.\cosh z$	2
(b) Find $\nabla.\overline{\mathrm{U}}$ and $\nabla\mathrm{x}\,\overline{\mathrm{U}}$ where : $\overline{U} = (x^2y)i + (yz)j + (xz + \cos z)k.$	4
(c) Find the integral : $\int_{(0,0)}^{(2,4)} (x+y)dx + (2x-y)dy$ along the curve $y = x^2$	4
Question 3	
(a)Find u and v of the function : $f(z) = \ln z$ and show that they satisfy Remmain's equations.	4
(b)If C is the circle | z | = 3, find the integrals : (i) $\oint_C \frac{\ln(z+8)}{z+4} dz$ (ii) $\oint_C \frac{\sqrt{3z+1}}{z} dz$ (iii) $\oint_C \frac{z}{(z-1)(z-2)^2} dz$	8
Question 4	
(a)Find u and v of the function : $f(z) = z^2 + 2z$ and show that u is harmonic.	3
(b)Find the Fourier series of the function : $f(x) = \mathrm{x}, \quad -1 \le \mathrm{x} \le 1, \quad \mathrm{f(x+2) = f(x)}$	3

Good Luck *Dr. Mohamed Eid*

Model Answer

Answer of Question 1

(a) $f_x = x.\cosh x + \sinh x, \quad f_y = 8y^3, \quad f_z = \frac{1}{z}$

---*4- Marks*

(b)Differentiate with respect to a, we get : $-2(y-a) \quad = 4$. Then $a = y + 2$

The envelope is : $x^2 + (2)^2 = 4(y+2)$ Or $x^2 = 4y + 4$

---*4- Marks*

(c)From : $f_x = 2x - 8 = 0, \quad f_y = -6y + 12$. We get x = 4, y = 2.

Then $\Delta = f_{xx} . f_{yy} - (f_{xy})^2 = (2)(-6) = -12$. Then (4, 2) is saddle point.

---*4- Marks*

Answer of Question 2

(a) $\nabla F = (4x^3 + \sin y)i + (x\cos y + \cosh z)j + (y\sinh z)k$

---*2- Marks*

(b) $\nabla . \bar{U} = 2xy + z + x - \sin z$.

$$\nabla \mathrm{x} \bar{\mathrm{U}} = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ \frac{\partial}{\partial \mathrm{x}} & \frac{\partial}{\partial \mathrm{y}} & \frac{\partial}{\partial \mathrm{z}} \\ \mathrm{x}^2\mathrm{y} & \mathrm{yz} & \mathrm{xz} + \cos \mathrm{z} \end{vmatrix} = (0 - \mathrm{y})\mathrm{i} - (\mathrm{z} - 0)\mathrm{j} + (0 - \mathrm{x}^2)\mathrm{k}$$

---*4- Marks*

(c) $\int_{(0,0)}^{(2,4)} (x+y)dx + (2x - y)dy = \int_0^2 (x + 5x^2 - 2x^3)\, dx = \frac{22}{3}$

---*4- Marks*

Answer of Question 3

(a) $f(z) = \ln z = \ln(x + iy) = \ln[\sqrt{x^2 + y^2}. e^{\mathrm{i}\tan^{-1}\frac{y}{x}}] = \frac{1}{2}\ln(x^2 + y^2) + \mathrm{i}\tan^{-1}\frac{y}{x}$

Then $u = \frac{1}{2}\ln(x^2 + y^2) \;\; and \;\; u_x = \frac{x}{x^2+y^2}, \; u_y = \frac{y}{x^2+y^2}$

$v = \tan^{-1}\frac{y}{x} \;\; and \;\; v_x = -\frac{y}{x^2+y^2}, \; v_y = \frac{x}{x^2+y^2}$

We see that $u_x = v_y \;\; and \;\; u_y = -v_x$

---*4- Marks* (b)(i)$\oint_C \frac{\ln(z+8)}{z+4}dz = 0$ because $\frac{\ln(z+8)}{z+4}$ is analytic inside C, – 4 outside C.

(ii) $\oint_C \frac{\sqrt{3z+1}}{z}dz = 2\pi i\, f(0) = 2\pi i\sqrt{0+1} = 2\pi i$ because 0 inside C.

(iii)Since the two points 1, 2 inside C. Then

$$\operatorname*{Res}_{z=1} f(z) = \lim_{z\to1}(z-1).\frac{z}{(z-1)(z-2)^2} = \lim_{z\to1}\frac{z}{(z-2)^2} = 1$$

$$\operatorname*{Res}_{z=2} f(z) = \lim_{z\to2}\left[(z-2)^2.\frac{z}{(z-1)(z-2)^2}\right]` = \lim_{z\to2}\left[\frac{z}{z-1}\right]` = -1$$

Then $\oint_C \frac{z}{(z-1)(z-2)^2}dz = 2\pi i\,(1-1) = 0$

--*8- Marks*

Answer of Question 4

(a) $f(z) = z^2 + 2z = 2x + x^2 - y^2 + i(2y + 2xy)$

Then $u = 2x + x^2 - y^2$ and $v = 2y + 2xy$

Then $u_{xx} + u_{yy} = 2 - 2 = 0$. Hence u is harmonic.

--*3- Marks*

(b)Since f(x) is odd. Then $a_0 = a_n = 0$

$$b_n = \frac{1}{1}\int_{-1}^{1} x.\sin n\pi x\,dx = 2\int_0^1 x.\sin n\pi x\,dx = 2\left[\frac{-x\cos n\pi x}{n\pi} + \frac{\sin n\pi x}{(n\pi)^2}\right] = \frac{2}{n\pi}(-1)^{n+1}$$

$$Then\quad f(x) = \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n\pi}\sin n\pi x$$

--*3- Marks*

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